Universalities in hadron production and the maximum entropy principle

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A shape of statistical momentum distribution of hadrons produced in high energy particle collisions closely resembles one observed for a broad variety of phenomena in nature. An attempt was made to understand a genesis of this distribution beyond the context of each particular phenomenon.

A multi-hadron production in high energy hadronic collisions is a complicated phenomenon generally modeled by a dynamic system of hadronizing quarks and gluons with many degrees of freedom and high level of correlations. The properties of produced hadrons at any given interaction cannot be predicted. But statistical properties, energy and momentum averages, correlation functions, and probability density functions show regular behavior. Thus, statistical methods must be applied to understand properties of particle production presented by the experimentally measured distributions.

A statistical model for computing high energy collisions of protons with multiple production of particles has been discussed first by E.Fermi in 1950 [1]. Later on, R.Hagedorn has proposed a statistical thermodynamic model to describe the momentum spectra of particles produced in $pp-$collisions [2]. This model approximates the experimentally measured exponential momentum spectra of hadrons with a Boltzmann-like statistical distribution. With an advance of high energy collision experiments with high statistic accumulated the measured momentum spectra are found to deviate from the exponential form. Namely, at high values of particle's transverse momentum (P_T) the spectrum shows a power-law behaviour. This observation is interpreted as a proof of the underlying QCD dynamics of hadronizing partonic system produced in the particle collisions. Though using QCD one calculates elementary interactions of quarks and gluons at microscopic level, a complexity of the system generally doesn't allow an extension of these calculations to predict the observed particle spectra. In addition, it is observed that the relative rates of different particle production and parameters of the momentum spectra depend on global particle's variables like the mass and spin [3, 4]. Therefore, it is tempting to reconsider

a statistical approach to understand the particle's transverse momentum spectra in the whole range.

Firstly, we note that the invariant particle production cross-section is approximated by a damped power-law function of a particle's transverse momentum:

$$\frac{d\sigma}{dydP_T^2} \sim \frac{1}{(a_0 + P_T)^n}\,, \tag{1}$$

where a_0 and n are parameters of the distribution function. For simplicity, we consider only particles produced at central rapidity plateau. These particles do not belong to the fragmentation regions of the colliding beams.

It is interesting to note, the same form of the damped power-law distribution describes a variety of phenomena like a velocity spectrum in turbulent liquid, ion energy distribution in geomagnetic plasma, a distribution of the earthquakes as function of it's magnitude (empirical Gutenberg-Richter law), distributions of the avalanches and landslides, forest fires and solar flares, rains and winds, distributions related to the human activities like earnings and settlements sizes, distribution of inter-trade intervals observed on stock markets, statistical distribution of a number of sexual partners, etc. These distributions arise because the same stochastic process is at work, and this process can be understood beyond the context of each example. Moreover, one can see that for small values of the distributed variable the damped power-law expression is reduced to the exponential statistical distribution. What is a genesis of the power-law and exponential statistical distributions?

The least biased method to obtain statistical distributions, which are realized in the nature was promoted by E.T.Jaynes as Maximum Entropy Principle [5]. This Principle states that the physical observable has a distribution, consistent with given constraints which maximizes the entropy. As an example, the Boltzmann exponential distribution arises naturally from a maximization of Gibbs-Shannon entropy under a constraint on an average value of the distributed variable. The textbook examples of the exponential distributions are the energy of the molecules of an ideal gas within an isolated volume and a distance between two neighboring points with $N-$points randomly spread over a limited interval with length L. Indeed, in both examples the average values of the distributed variable are well defined. Namely, due to a conservation of the number of molecules and the total energy in the gas volume their ratio gives a predefined value of average kinetic energy per molecule of gas. Similarly, an average distance between two points is obviously defined by a ratio of $L/(N-1)$. However, generally speaking, the

constraint on the average value in most cases has a little sense. Therefore, one needs another suitable form of a constraint which results in the damped power-law distribution while maximizing the Gibbs-Shannon entropy [1]. This new constraint has to correspond to a new conservation law. There is no obvious known conservation law which could yield the damped power-law distribution. Therefore, we propose here a toy model. In this model each measurement of a physical value x_i of interest corresponds to a gain of some bits of information equal to $ln(a_0 + x_i)$. Assume, that an observer is allowed to gain in a set of measurements in average a limited information only. This assumption corresponds to the following constraint

$$\Sigma P_i ln(a_0 + x_i) = const\,, \tag{2}$$

where P_i is a probability to observe a value x_i. Maximization of entropy under the constraint (2) results in the damped power-law probability distribution as function of x_i. It is interesting to note that the above constraint (2) is reduced to a geometric mean value of terms $(a_0 + x_i)$. This makes different events correlated with a correlation strength increasing for smaller a_0 values. A conservation of an average information is lively exemplified in a world of fractals. An observer doesn't gain new information about the fractal object structure by changing a scale of resolution. If so, a meaning of the a_0 parameter is to define a minimal available information. It is interesting to note, the experimental data show that for hadron transverse momentum spectra the a_0 value is close to the mass of produced hadron [4]. In addition to the direct measurements of particle spectra a particle-particle correlation and a broadening of non-Poisson particle multiplicity distribution are likely intrinsically related to the appearance of power-law tails in particle momentum spectra. Therefore, an analysis of their dynamics for varying interaction energies will allow further progress. This makes high statistics data accumulated in high energy particle interactions to be a unique laboratory to study the whole class of the power-law phenomena.

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[1] For simplicity here we stick to the Gibbs-Shannon entropy form only. Postulating other forms of entropy, like one used in non-extensive thermodynamics [6] reveals additional conceptual problems

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